We'll do chapter 5 in a different order from in the book.

Monomials are smaller than polynomials, so we'll do all the monomial parts first

5.2
5.4
5.6

Then we'll do the polynomials

5.1
5.3
5.5

Exponents of Monomials

Objectives: i) Simplify expressions with monomials
- multiply monomials
- monomials raised to a power

Definition: A Monomial is the product of a constant and one or more variables, each raised to a whole number exponent

[Recall: $\mathbb{W}$ = whole numbers = $\{0, 1, 2, 3 \ldots\}$]

there is no + or − (addition or subtraction) in a monomial.

Simplify and write with exponents. (For now, even the coefficients.)

① $2^3 \cdot 2^4 = \underbrace{2\cdot2\cdot2}\cdot\underbrace{2\cdot2\cdot2\cdot2} = \boxed{2^7} = 128$

same base 2
add exponents $3+4=7$

② $x^3 \cdot x^4 = \underbrace{x\cdot x\cdot x}\cdot\underbrace{x\cdot x\cdot x\cdot x} = \boxed{x^7}$

same base x
add exponents $3+4=7$

③ $(-3)^2(-3) = \underbrace{(-3)(-3)}\underbrace{(-3)} = \boxed{(-3)^3} = -27$

same base -3
add exponents $2+1=3$

Exponent means repeated multiplication

④ $m^3 \cdot m^5 \cdot n^9 = \underbrace{m^3 \cdot m^5}_{\text{same base}} \cdot \underbrace{n^9}_{\text{different base}}$

$= m^{3+5} \cdot n^9$

$= \boxed{m^8 \cdot n^9}$

Exponent Law #1
"Product Rule"
$a^m \cdot a^n = a^{m+n}$
same base
add exponents

⑤ $(2^3)^5 = \underbrace{2^3 \cdot 2^3 \cdot 2^3 \cdot 2^3 \cdot 2^3}$

$= 2^{3+3+3+3+3}$

$= 2^{3\cdot5}$

$= \boxed{2^{15}} = 32768$

⑯ $(x^3)^5 = x^{3\cdot5} = \boxed{x^{15}}$

even exponent: positive result

⑦ $[(-4)^2]^3 = (-4)^{2\cdot3} = (-4)^6 = \boxed{4^6} = 4096$

⑧ $[(-n)^3]^5 = (-n)^{3\cdot5} = (-n)^{15} = \boxed{-n^{15}}$

Exponent Law #2
"Power Rule"
$(a^m)^n = a^{m\cdot n}$
one base
mult exponents

⑨ $(3b)^4 = 3b \cdot 3b \cdot 3b \cdot 3b$

$= 3^4 \cdot b^4$

$= \boxed{3^4 \cdot b^4} = 81b^4$

Math 45 5.2

Exponent Law #3
"Product to power Rule"
$(a \cdot b)^n = a^n \cdot b^n$

Exponent Law #4
"Quotient to Power Rule"
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

You may wish to think of these as the "sharing rule"
⇒ share the exponent outside ( ) on every part inside ( ).

(10) $\left(\frac{-2}{3}\right)^4 = \frac{(-2)^4}{3^4} = \boxed{\frac{16}{81}}$

(11) $\left(-\frac{2}{3}a^2b\right)^3 = \frac{(-2)^3}{(3)^3} \cdot (a^2)^3 \cdot (b)^3 = \frac{-8}{27}a^{2\cdot 3}b^3 = \boxed{\frac{-8}{27}a^6b^3}$

Exponent Law #5
"Quotient Rule"
$\frac{a^n}{a^m} = a^{n-m}$

- same base
- subtract exponents
- num exp − denom exp means result in num
- denom exp − num exp means result in denom

(12) $\frac{n^6}{n^2} = \frac{n^{6-2}}{1} = \boxed{n^4}$

(13) $\frac{n^2}{n^6} = \frac{n^{2-6}}{1} = \frac{1}{n^{6-2}} = \boxed{\frac{1}{n^4}}$ if positive exp required.

$\downarrow$

$\boxed{n^{-4}}$ if neg. exponents permitted

Math 45 5.2

Simplify completely.

This means
- no parentheses
- all coefficients combined
- all fractions reduced
- all like bases combined.

Read instructions — you may be required to use only positive exponents in your results.

(14) $(-3x^2y)(-7x^5y^3)$

$= (-3)(-7)(x^2)(x^5)(y)(y^3)$

$= +21x^{2+5}y^{1+3}$

$= \boxed{21x^7y^4}$

everything is multiplied, the negative is inside ( ), so it's not subtract.

(15) $(-2m^2n^3)^4$

$= (-2)^4(m^2)^4(n^3)^4$

$= 16m^{2\cdot4}n^{3\cdot4}$

$= \boxed{16m^8n^{12}}$

step 1: Resolve all ( )
step 2: Write like bases together
step 3: Combine like bases

(16) $\left(\frac{4}{5}q\right)^2(-5q)^3\left(\frac{1}{4}q^4\right)$

↑ exp is outside parentheses

↑ exp is outside parentheses

↑ exp is inside parentheses — does not belong to $\frac{1}{4}$.

$= \frac{4^2}{5^2}\cdot q^2\cdot(-5)^3\cdot q^3\cdot\frac{1}{4}\cdot q^4$

$= \frac{16}{25}q^2\cdot(-125)q^3\cdot\frac{1}{4}\cdot q^4$

$= \frac{\cancel{16}^{4}}{\cancel{25}}\cdot\frac{(-\cancel{125}^{5})}{\cancel{4}_{1}}\cdot q^2\cdot q^3\cdot q^4$

$= 4(-5)q^{2+3+4}$

$= \boxed{-20q^9}$

Simplify completely.

(17) $-4(-7xy^3)\left(\frac{3}{7}x^3y\right)^2$

$= (-4)\cdot(-7)\cdot x\cdot y^3\cdot\frac{3^2}{7^2}\cdot(x^3)^2\cdot y^2$ ← sharing rule (product to power rule)

$= 28\cdot x\cdot y^3\cdot\frac{9}{49}\cdot x^{3\cdot2}\cdot y^2$ ← power rule

$= \overset{4}{\cancel{28}}\cdot\frac{9}{\cancel{49}_{7}}\cdot x\cdot x^6\cdot y^3\cdot y^2$ ← write like bases next to each other

$= \frac{36}{7}x^{1+6}y^{3+2}$ ← product rule

$= \boxed{\frac{36}{7}x^7y^5}$

(15) $(9t)^2\left(\frac{4}{3}t^2\right)^3\left(\frac{1}{3}t^5\right)$

$= 9^2\cdot t^2\cdot\frac{4^3}{3^3}\cdot(t^2)^3\cdot\frac{1^0}{3^0}\cdot t^5$

$= 81\cdot t^2\cdot\frac{64}{27}\cdot t^{2\cdot3}\cdot\frac{1}{3}\cdot t^5$ ← power rule

$= \overset{27}{\cancel{81}}\cdot\frac{64}{\cancel{27}}\cdot\frac{1}{\cancel{3}_{1}}\cdot t^2\cdot t^6\cdot t^5$ ← write like bases next to each other

$= 64\cdot t^{2+6+5}$

$= \boxed{64t^{13}}$

Step 1: Resolve all parentheses using the sharing rule and the power rule.

Step 2: Write like bases next to each other

Step 3: Combine like bases using the product rule.

Student: _____	**Instructor:** Martha Carey	**Assignment:** 5.2
Date: _____	**Course:** Math 45-26 Sp13	
Time: _____	**Book:** Sullivan: Elementary & Intermediate Algebra, 2e	

5.2.79. Simplify the expression.

$$(-xy)(x^3y)(-3x)^3$$

$(-xy)(x^3y)(-3x)^3 = \square$

(Simplify your answer. Type your answer using exponential notation.)

Resolve parentheses:

$$(-xy)(x^3y)(-3x)^3$$

⇑ only this set requires sharing rule

$$= -x \cdot y \cdot x^3 \cdot y \cdot (-3)^3 \cdot x^3$$

Write like bases next to each other:

$$= (-3)^3 \cdot (-x) \cdot x^3 \cdot x^3 \cdot y \cdot y$$

simplify $(-3)^3$

$$= (-27)(-x)\, x^3 x^3 \cdot y \cdot y$$

simplify sign

$$= +27 \cdot x \cdot x^3 \cdot x^3 \cdot y \cdot y$$

Combine like bases by adding exponents (product rule)

$$= 27\, x^{1+3+3} \cdot y^{1+1}$$

$$= \boxed{27x^7y^2}$$

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Date: _____	**Course:** Math 45-26 Sp13	
Time: _____	**Book:** Sullivan: Elementary & Intermediate Algebra, 2e	

5.2.87. Simplify the expression completely.

$(-5x)^2(4x^2)^2$

$(-5x)^2(4x^2)^2 = \square$

(Simplify your answer. Use integers or fractions for any numbers in the expression.)

Resolve ( ).

$(-5x)^2(4x^2)^2$

Both sets of ( ) have exponents outside.

$= (-5)^2 \cdot x^2 \cdot 4^2 \cdot (x^2)^2$

power rule (multiply exponents)

$= 25 \cdot 16 \cdot x^2 \cdot x^4$ collect like bases

$= 400\, x^{2+4}$ add exponents

$= \boxed{400x^6}$

Student: _____	**Instructor:** Martha Carey	**Assignment:** 5.2
Date: _____	**Course:** Math 45-26 Sp13	
Time: _____	**Book:** Sullivan: Elementary & Intermediate Algebra, 2e	

5.2.91. Simplify the expression.

$$(3x)^3(-2x)^2\left(\frac{1}{3}x^3\right)^3$$

$(3x)^3(-2x)^2\left(\frac{1}{3}x^3\right)^3 = \square$

(Simplify your answer. Type your answer using exponential notation.)

Resolve ( )

$(3x)^3(-2x)^2\left(\frac{1}{3}x^3\right)^3$

all 3 sets of parentheses have exp outside

$= 3^3 \cdot x^3 \cdot (-2)^2 \cdot x^2 \cdot \left(\frac{1}{3}\right)^3 \cdot (x^3)^3$

$= 27 \cdot x^3 \cdot 4 \cdot x^2 \cdot \frac{1}{27} \cdot x^{3\cdot 3}$ ← power rule, multiply exp.

$= 27 \cdot 4 \cdot \frac{1}{27} \cdot x^3 \cdot x^2 \cdot x^9$

$= 4 \cdot x^{3+2+9}$ Product rule

$= \boxed{4x^{14}}$

Student: _____	**Instructor:** Martha Carey	**Assignment:** 5.2
Date: _____	**Course:** Math 45-26 Sp13	
Time: _____	**Book:** Sullivan: Elementary & Intermediate Algebra, 2e	

5.2.93. Simplify the expression.

$$-2(-3xy^2)^2\left(\frac{2}{3}x^2y\right)^3$$

$$-2(-3xy^2)^2\left(\frac{2}{3}x^2y\right)^3 = \square$$

(Simplify your answer. Use integers or fractions for any numbers in the expression. Type your answer using exponential notation.)

Resolve ( )

$$-2(-3xy^2)^2\left(\frac{2}{3}x^2y\right)^3$$

both sets of ( ) have exp outside

$$= -2 \cdot (-3)^2 \cdot x^2 \cdot (y^2)^2 \cdot \left(\frac{2}{3}\right)^3 \cdot (x^2)^3 \cdot y^3$$

$$= (-2)(-27) \cdot x^2 \cdot y^{2\cdot 2} \cdot \frac{2^3}{3^3} \cdot x^{2\cdot 3} \cdot y^3$$

mult exponents.

$$= 54\, x^2 y^4 \cdot \frac{8}{27} \cdot x^6 y^3$$

$$= \cancel{54}^{2} \cdot \frac{8}{\cancel{27}_{1}} \cdot x^2 \cdot x^6 \cdot y^4 \cdot y^3$$ collect

$$= 16\, x^{2+6} \cdot y^{4+3}$$

$$= \boxed{16x^8y^7}$$

Student: _____	**Instructor:** Martha Carey	**Assignment:** 5.2
Date: _____	**Course:** Math 45-26 Sp13	
Time: _____	**Book:** Sullivan: Elementary & Intermediate Algebra, 2e	

5.2.37. Simplify the expression.

$[(-z)^4]^5$

$[(-z)^4]^5 = (-z)^{20}$ (Simplify your answer. Type your answer using exponential notation.)

or z^{20} both accepted by MXL

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Date: _____	**Course:** Math 45-26 Sp13	
Time: _____	**Book:** Sullivan: Elementary & Intermediate Algebra, 2e	

5.2.31. Simplify the expression.

$(-x)^{11}(-x)^2$

$(-x)^{11}(-x)^2 = (-x)^{13}$

(Simplify your answer. Type your answer using exponential notation.)

$= -x^{13}$ both accepted by MXL